INTEGRALI

1. Izračunajte $\int \left(x^2 + \sqrt{x} + \sqrt[3]{x}\right) dx$

   Rješenje:

$$\int \left(x^2 + \sqrt{x} + \sqrt[3]{x}\right) dx = \int x^2 dx + \int \sqrt{x} dx + \int \sqrt[3]{x} dx =$$
$$= \int x^2 dx + \int x^{\frac{1}{2}} dx + \int x^{\frac{1}{3}} dx =$$
$$= \frac{x^{2+1}}{2+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C =$$
$$= \frac{x^3}{3} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + C =$$
$$= \frac{x^3}{3} + \frac{2}{3}x^{\frac{3}{2}} + \frac{3}{4}x^{\frac{4}{3}} + C$$

2. Izračunajte $\int(3x^2 - \frac{1}{3\sqrt{x}})dx$.

Rješenje:

$$\int(3x^2 - \frac{1}{3\sqrt{x}})dx = \int 3x^2dx - \int \frac{1}{3\sqrt{x}}dx =$$
$$= 3\int x^2dx - \frac{1}{3}\int \frac{1}{\sqrt{x}}dx =$$
$$= 3\int x^2dx - \frac{1}{3}\int \frac{1}{x^{\frac{1}{2}}}dx =$$
$$= 3\int x^2dx - \frac{1}{3}\int x^{\frac{-1}{2}}dx =$$
$$= 3 \cdot \frac{x^{2+1}}{2+1} - \frac{1}{3} \cdot \frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + C =$$
$$= 3 \cdot \frac{x^3}{3} - \frac{1}{3} \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C =$$
$$= x^3 - \frac{1}{3} \cdot 2x^{\frac{1}{2}} + C =$$
$$= x^3 - \frac{2}{3}x^{\frac{1}{2}} + C$$

3. Izračunajte $\int(4x+\frac{1}{\sqrt{x}})dx$.

Rješenje:

$$\int(4x+\frac{1}{\sqrt{x}})dx=\int 4xdx+\int\frac{1}{\sqrt{x}}dx=$$
$$=4\int xdx+\int\frac{1}{x^{\frac{1}{2}}}dx=4\int xdx+\int x^{\frac{-1}{2}}dx=$$
$$=4\cdot\frac{x^{1+1}}{1+1}+\frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}+C=$$
$$=4\cdot\frac{x^2}{2}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C=$$
$$=2x^2+2x^{\frac{1}{2}}+C=$$
$$=2x^2+2\sqrt{x}+C$$

4. Izračunajte $\int(-x^3+\sqrt{x}+x^{\frac{1}{3}})dx$.

Rješenje:

$$\begin{aligned}
\int(-x^3+\sqrt{x}+x^{\frac{1}{3}})dx &= \int -x^3dx+\int\sqrt{x}dx+\int x^{\frac{1}{3}}dx = \\
&= -\int x^3dx+\int x^{\frac{1}{2}}dx+\int x^{\frac{1}{3}}dx = \\
&= -\frac{x^{3+1}}{3+1}+\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}+C = \\
&= -\frac{x^4}{4}+\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{4}{3}}}{\frac{4}{3}}+C = \\
&= -\frac{x^4}{4}+\frac{2}{3}x^{\frac{3}{2}}+\frac{3}{4}x^{\frac{4}{3}}+C = \\
&= -\frac{x^4}{4}+\frac{2}{3}x\sqrt{x}+\frac{3}{4}x^{\frac{4}{3}}+C
\end{aligned}$$

5. Izračunajte $\int 4xlnxdx$.

Rješenje:

$$\int 4xlnxdx = \begin{bmatrix} u = lnx & dv = 4xdx \\ du = \frac{1}{x}dx & v = \int 4xdx = 2x^2 \end{bmatrix} =$$

$$= u \cdot v - \int vdu = lnx \cdot 2x^2 - \int 2x^2 \cdot \frac{1}{x}dx =$$

$$= 2x^2lnx - \int 2xdx = 2x^2lnx - 2\int xdx =$$

$$= 2x^2lnx - 2 \cdot \frac{x^2}{2} + C = 2x^2lnx - x^2 + C$$

7. Izračunajte $\int \frac{2x-2}{x^2-2x+9}dx$.

Rješenje:

$$\int \frac{2x-2}{x^2-2x+9}dx = \begin{bmatrix} t = x^2-2x+9 \\ dt = (2x-2)dx \end{bmatrix} =$$
$$= \int \frac{dt}{t} = ln|t| + C = ln|x^2-2x+9| + C$$

8. Izračunajte $\int \frac{lnx}{x}dx$.

Rješenje:

$$\int \frac{lnx}{x}dx = \begin{bmatrix} t = lnx \\ dt = \frac{1}{x}dx \end{bmatrix} =$$
$$= \int tdt = \frac{t^2}{2} + C = \frac{ln^2x}{2} + C$$

9. Izračunajte $\int xe^{x^2}dx$.

Rješenje:

$$\int xe^{x^2}dx = \begin{bmatrix} t = x^2 \\ dt = 2xdx / : 2 \\ \frac{dt}{2} = xdx \end{bmatrix} = \int e^t \frac{dt}{2} =$$
$$= \frac{1}{2}\int e^t dt = \frac{1}{2}e^t + C = \frac{1}{2}e^{x^2} + C$$

10. Izračunajte $\int 6x^2e^{x^3}dx$.

Rješenje:

$$\int 6x^2e^{x^3}dx = \begin{bmatrix} t = x^3 \\ dt = 3x^2dx / \cdot 2 \\ 2dt = 6x^2dx \end{bmatrix} =$$
$$= \int 2e^t dt = 2e^t + C = 2e^{x^3} + C$$