346. Izračunajte koeficijent smjera sekante parabole

$$y = 2x - x^2,$$

ako su apscise presječnih tačaka:

a) $x_1 = 1, \quad x_2 = 2;$

b) $x_1 = 1, \quad x_2 = 0{,}9;$

c) $x_1 = 1, \quad x_2 = 1 + h.$

Kojem limesu teži koeficijent smjera sekante u posljednjem primjeru, ako $h \to 0$?

347. Koja je srednja brzina mijenjanja funkcije $y = x^3$ u granicama $1 \leqslant x \leqslant 4$?

348. Zakon gibanja tačke je $s = 2t^2 + 3t + 5$, gdje je put s zadan u centimetrima, a vrijeme t u sekundama. Kolika je srednja brzina tačke u vremenu ode $t = 1$ do $t = 5$?

349. Izračunajte srednji uspon krivulje $y = 2^x$ u intervalu $1 \leqslant x \leqslant 5$.

350. Izračunajte srednji uspon krivulje $y = f(x)$ u intervalu $[x, \ x + \Delta x]$.

351. Što se razumijeva pod usponom krivulje $y = f(x)$ u zadanoj tački x?

352. Definirajte: a) srednju brzinu vrtnje, b) trenutnu brzinu vrtnje.

353. Zagrijano tijelo smješteno je u okolicu niže temperature i hladi se. Šta znači: a) srednja brzina ohlađivanja; b) brzina ohlađivanja u danom trenutku?

354. Šta se razumijeva pod brzinom reagiranja tvari u kemijskoj reakciji?

355. Neka je $m = f(x)$ masa nehomogene šipke na dijelu $[0, x]$. Što se razumijeva pod: a) srednjom linearnom gustoćom šipke na dijelu $[x, x + \Delta x]$; b) linearnom gustoćom šipke u tački x?

356. Izračunajte kvocijent $\dfrac{\Delta y}{\Delta x}$ za funkciju $y = \dfrac{1}{x}$ u tački $x = 2$, ako je:

a) $\Delta x = 1$; b) $\Delta x = 0{,}1$; c) $\Delta x = 0{,}01$.

Kolika je derivacija y' za $x = 2$?

357.** Izračunajte derivaciju funkcije $y = \operatorname{tg} x$.

358. Izračunajte $y' = \lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x}$ za funkciju:

a) $y = x^3$; c) $y = \sqrt{x}$;

b) $y = \dfrac{1}{x^2}$; d) $y = \operatorname{ctg} x$.

359.** Izračunajte $f'(8)$, ako je $f(x) = \sqrt[3]{x}$.

360. Nadite $f'(0)$, $f'(1)$, $f'(2)$, ako je $f(x) = x(x-1)^2(x-2)^3$.

361.* U kojim tačkama se derivacija funkcije $f(x) = x^3$ brojčano podudara sa vrijednošću same funkcije, tj. kada je $f(x) = f'(x)$?

362. Zakon gibanja tačke je $s = 5t^2$, pri čemu je put s zadan u metrima, a vrijeme t u sekundama. Izračunajte brzinu gibanja u trenutku $t = 3$.

363. Nađite koeficijent smjera tangente na krivulju $y = 0{,}1\,x^3$ u tački kojoj je apscisa $x = 2$.

364. Nađite koeficijent smjera tangente na krivulju $y = \sin x$ u tački $(\pi; 0)$.

365. Izračunajte vrijednost derivacije funkcije $f(x) = \dfrac{1}{x}$ u tački $x = x_0$ $(x_0 \neq 0)$.

366*. Koliki su koeficijenti smjerova tangenata na krivulje $y = \dfrac{1}{x}$ i $y = x^2$ u njihovu sjecištu? Izračunajte kut koji zatvaraju te tangente.

367.** Pokažite da ove funkcije nemaju konačnih derivacija u zadanim tačkama:

a) $y = \sqrt[3]{x^2}$ u tački $x = 0$;

b) $y = \sqrt[5]{x-1}$ u tački $x = 1$;

c) $y = |\cos x|$ u tačkama $x = \dfrac{2k+1}{2}\pi \quad (k = 0, \pm 1, \pm 2, \ldots)$.

2. Tablično deriviranje

1°. Osnovna pravila deriviranja. Ako je c konstanta, a $u = \varphi(x)$, $v = \psi(x)$ su funkcije koje imaju derivacije, onda je

1) $(c)' = 0$;

2) $(x)' = 1$;

3) $(u \pm v)' = u' \pm v'$;

4) $(cu)' = cu'$;

5) $(uv)' = u'v + v'u$;

6) $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - v'u}{v^2} \quad (v \neq 0)$;

7) $\left(\dfrac{c}{v}\right)' = -\dfrac{cv'}{v^2} \quad (v \neq 0)$.

2°. Tablica derivacija osnovnih funkcija

I. $(x^n)' = nx^{n-1}$.

II. $(\sqrt{x})' = \dfrac{1}{2\sqrt{x}} \quad (x > 0)$.

III. $(\sin x)' = \cos x$.

IV. $(\cos x)' = -\sin x$.

V. $(\operatorname{tg} x)' = \dfrac{1}{\cos^2 x}$.

VI. $(\operatorname{ctg} x)' = -\dfrac{1}{\sin^2 x}$

VII. $(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$.

VIII. $(\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$.

IX. $(\operatorname{arctg} x)' = \dfrac{1}{1+x^2}$.

X. $(\operatorname{arcctg} x)' = -\dfrac{1}{x^2+1}$.

XI. $(a^x)' = a^x \ln a \quad (a > 0)$.

XII. $(e^x)' = e^x$.

XIII. $(\ln x)' = \dfrac{1}{x} \quad (x>0)$.

XIV. $(\log_a x)' = \dfrac{1}{x \ln a} = \dfrac{\log_a e}{x} \quad (x>0,\ a>0)$.

XV. $(\operatorname{sh} x)' = \operatorname{ch} x$.

XVI. $(\operatorname{ch} x)' = \operatorname{sh} x$.

XVII. $(\operatorname{th} x)' = \dfrac{1}{\operatorname{ch}^2 x}$.

XVIII. $(\operatorname{cth} x)' = -\dfrac{1}{\operatorname{sh}^2 x}$.

XIX. $(\operatorname{Arsh} x)' = \dfrac{1}{\sqrt{1+x^2}}$.

XX. $(\operatorname{Arch} x)' = \dfrac{1}{\sqrt{x^2-1}} \quad (x>1)$.

XIX. $(\operatorname{Arth} x)' = \dfrac{1}{1-x^2} \quad (|x|<1)$.

XXII. $(\operatorname{Arcth} x)' = -\dfrac{1}{x^2-1} \quad (|x|>1)$.

3°. Pravilo deriviranja složenih funkcija. Ako je $y=f(u)$ i $u=\varphi(x)$ tj. $y=f[\varphi(x)]$, a funkcije y i u imaju derivacije, tada je

$$y'_x = y'_u u'_x \tag{1}$$

ili drukčije pisano

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

To pravilo primjenjuje se na »lanac« bilo kojega konačnog broja funkcija koje deriviramo.

Primjer 1. Izračunajmo derivaciju funkcije

$$y = (x^2 - 3x + 3)^5.$$

Rješenje. Stavivši $y=u^5$, pri čemu je $u=x^2-2x+3$, Imat ćemo prema (1):

$$y' = (u^5)'_u (x^2 - 2x + 3)'_x = 5u^4 (2x - 2) = 10 (x - 1) (x^2 - 2x + 3)^4.$$

Primjer 2. Izračunajmo derivaciju funkcije

$$y = \sin^3 4x.$$

Rješenje. Stavivši

$$y = u^3; \quad u = \sin v; \quad v = 4x,$$

dobivamo

$$y' = 3u^2 \cdot \cos v \cdot 4 = 12 \sin^2 4x \cos 4x.$$

Izračunajte derivacije ovih funkcija (u br. 368 do 408 ne primjenjuje se pravilo deriviranja složenih funkcija):

A. Algebarske funkcije

368. $y = x^5 - 4x^3 + 2x - 3$.

369. $y = \dfrac{1}{4} - \dfrac{1}{3}x + x^2 - 0{,}5x^4$.

370. $y = ax^2 + bx + c$.

371. $y = -\dfrac{5x^3}{a}$.

372. $y = at^m + bt^{m+n}$.

373. $y = \dfrac{ax^6 + b}{\sqrt{a^2+b^2}}$.

374. $y = \dfrac{\pi}{x} + \ln 2$.

375. $y = 3x^{\frac{2}{3}} - 2x^{\frac{5}{2}} + x^{-3}$.

376*. $y = x^2 \sqrt[3]{x^2}$.

377. $y = \dfrac{a}{\sqrt[3]{x^2}} - \dfrac{b}{x\sqrt[3]{x}}$.

378. $y = \dfrac{a+bx}{c+dx}$.

379. $y = \dfrac{2x+3}{x^2-5x+5}$.

380. $y = \dfrac{2}{2x-1} - \dfrac{1}{x}$.

381. $y = \dfrac{1+\sqrt{z}}{1-\sqrt{z}}$.

B. Trigonometrijske i arkus-funkcije

382. $y = 5 \sin x + 3 \cos x$.

383. $y = \operatorname{tg} x - \operatorname{ctg} x$.

384. $y = \dfrac{\sin x + \cos x}{\sin x - \cos x}$.

385. $y = 2t \sin t - (t^2 - 2) \cos t$.

386. $y = \operatorname{arctg} x + \operatorname{arcctg} x$.

387. $y = x \operatorname{ctg} x$.

388. $y = x \arcsin x$.

389. $y = \dfrac{(1+x^2)\operatorname{arctg} x - x}{2}$.

C. Eksponencijalne i logaritamske funkcije

390. $y = x^7 \cdot e^x$.

391. $y = (x-1)e^x$.

392. $y = \dfrac{e^x}{x^2}$.

393. $y = \dfrac{x^5}{e^x}$.

394. $f(x) = e^x \cos x$.

395. $y = (x^2 - 2x + 2)e^x$.

396. $y = e^x \arcsin x$.

397. $y = \dfrac{x^2}{\ln x}$.

398. $y = x^3 \ln x - \frac{x^3}{3}$.

399. $y = \frac{1}{x} + 2\ln x - \frac{\ln x}{x}$.

400. $y = \ln x \lg x - \ln a \log_a x$.

D. Hiperbolne i area-funkcije

401. $y = x \operatorname{sh} x$.

402. $y = \frac{x^2}{\operatorname{ch} x}$.

403. $y = \operatorname{th} x - x$.

404. $y = \frac{3 \operatorname{cth} x}{\ln x}$.

405. $y = \operatorname{arctg} x - \operatorname{Arth} x$.

406. $y = \arcsin x \operatorname{Arsh} x$.

407. $y = \frac{\operatorname{Arch} x}{x}$.

408. $y = \frac{\operatorname{Arcth} x}{1-x^2}$.

E. Složene funkcije

Izračunajte derivacije ovih funkcija (u br. 409 do 466 treba upotrijebiti pravilo deriviranja složenih funkcija s jednim međuargumentom):

409. $y = (1+3x-5x^2)^{30}$.

Rješenje. Označimo da je $1+3x-5x^2 = u$; tada je $y = u^{30}$. Dobivamo:

$$y'_u = 30u^{29}, \quad u'_x = 3-10x;$$
$$y'_x = 30\,u^{29}\cdot(3-10x) = 30\,(1+3x-5x^2)^{29}\cdot(3-10x).$$

410. $y = \left(\frac{ax+b}{c}\right)^3$.

411. $f(y) = (2a+3by)^2$.

412. $y = (3+2x^2)^4$.

413. $y = \frac{3}{56(2x-1)^7} - \frac{1}{24(2x-1)^6} - \frac{1}{40(2x-1)^5}$.

414. $y = \sqrt{1-x^2}$.

415. $y = \sqrt[3]{a+bx^3}$.

416. $y = (a^{2/3} - x^{2/3})^{3/2}$.

417. $y = (3-2\sin x)^5$.

Rješenje. $y' = 5\,(3-2\sin x)^4\cdot(3-2\sin x)' = 5\,(3-2\sin x)^4\cdot(-2\cos x) = -10\cos x\,(3-2\sin x)^4$.

418. $y = \operatorname{tg} x - \frac{1}{3}\operatorname{tg}^3 x + \frac{1}{5}\operatorname{tg}^5 x$.

419. $y = \sqrt{\operatorname{ctg} x} - \sqrt{\operatorname{ctg} \alpha}$.

420. $y = 2x + 5\cos^3 x$.

421*. $x = \operatorname{cosec}^2 t + \sec^2 t$.

422. $f(x) = -\frac{1}{6(1-3\cos x)^2}$.

423. $y = \frac{1}{3\cos^3 x} - \frac{1}{\cos x}$.

424. $y = \sqrt{\frac{3\sin x - 2\cos x}{5}}$.

425. $y = \sqrt[3]{\sin^2 x} + \frac{1}{\cos^3 x}$.

426. $y = \sqrt{1+\arcsin x}$.

427. $y = \sqrt{\operatorname{arctg} x} - (\arcsin x)^3$.

428. $y = \frac{1}{\operatorname{arctg} x}$.

429. $y = \sqrt{xe^x + x}$.

430. $y = \sqrt[3]{2e^x - 2^x + 1} + \ln^5 x$.

431. $y = \sin 3x + \cos\frac{x}{5} + \operatorname{tg}\sqrt{x}$.

Rješenje. $y' = \cos 3x\cdot(3x)' - \sin\frac{x}{5}\left(\frac{x}{5}\right)' + \frac{1}{\cos^2\sqrt{x}}(\sqrt{x})' = 3\cos 3x - \frac{1}{5}\sin\frac{x}{5} + \frac{1}{2\sqrt{x}\cos^2\sqrt{x}}$.

432. $y = \sin(x^2-5x+1) + \operatorname{tg}\frac{a}{x}$.

433. $f(x) = \cos(\alpha x + \beta)$.

434. $f(t) = \sin t \sin(t+\varphi)$.

435. $y = \frac{1+\cos 2x}{1-\cos 2x}$.

436. $f(x) = a \operatorname{ctg}\frac{x}{a}$.

437. $y = -\frac{1}{20}\cos(5x^2) - \frac{1}{4}\cos x^2$.

438. $y = \arcsin 2x$.

Rješenje. $y' = \frac{1}{\sqrt{1-(2x)^2}}\cdot(2x)' = \frac{2}{\sqrt{1-4x^2}}$.

439. $y = \arcsin\frac{1}{x^2}$.

440. $f(x) = \arccos\sqrt{x}$.

441. $y = \operatorname{arctg}\frac{1}{x}$.

442. $y = \operatorname{arcctg}\frac{1+x}{1-x}$.

443. $y = 5e^{-x^2}$.

444. $y = \frac{1}{5^{x^2}}$.

445. $y = x^2 10^{2x}$.

446. $f(t) = t\sin 2^t$.

447. $y = \arccos e^x$.

448. $y = \ln(2x+7)$.

449. $y = \lg\sin x$.

450. $y = \ln(1-x^2)$.

451. $y = \ln^2 x - \ln(\ln x)$.

452. $y = \ln(e^x + 5\sin x - 4\arcsin x)$.

453. $y = \operatorname{arctg}(\ln x) + \ln(\operatorname{arctg} x)$.

454. $y = \sqrt{\ln x + 1} + \ln(\sqrt{x}+1)$.

F. Razne funkcije

455.** $y = \sin^3 5x \cos^2 \frac{x}{3}$.

456. $y = -\frac{11}{2(x-2)^2} - \frac{4}{x-2}$.

457. $y = -\frac{15}{4(x-3)^4} - \frac{10}{3(x-3)^3} - \frac{1}{2(x-3)^2}$.

458. $y = \frac{x^8}{8(1-x^2)^4}$.

459. $y = \frac{\sqrt{2x^2-2x+1}}{x}$.

460. $y = \frac{x}{a^2\sqrt{a^2+x^2}}$.

461. $y = \frac{x^3}{3\sqrt{(1+x^2)^3}}$.

462. $y = \frac{3}{2}\sqrt[3]{x^2} + \frac{18}{7}x\sqrt[6]{x} + \frac{9}{5}x\sqrt[3]{x^2} + \frac{6}{13}x^2\sqrt[6]{x}$.

463. $y = \frac{1}{8}\sqrt[3]{(1+x^3)^8} - \frac{1}{5}\sqrt[3]{(1+x^3)^5}$.

464. $y = \frac{4}{3}\sqrt[4]{\frac{x-1}{x+2}}$.

465. $y = x^4(a-2x^3)^2$.

466. $y = \left(\frac{a+bx^n}{a-bx^n}\right)^m$.

467. $y = \frac{9}{5(x+2)^5} - \frac{3}{(x+2)^4} + \frac{2}{(x+2)^3} - \frac{1}{2(x+2)^2}$.

468. $y = (a+x)\sqrt{a-x}$.

469. $y = \sqrt{(x+a)(x+b)(x+c)}$.

470. $z = \sqrt[3]{y+\sqrt{y}}$.

471. $f(t) = (2t+1)(3t+2)\sqrt[3]{3t+2}$.

472. $x = \frac{1}{\sqrt{2ay-y^2}}$.

473. $y = \ln(\sqrt{1+e^x}-1) - \ln(\sqrt{1+e^x}+1)$.

474. $y = \frac{1}{15}\cos^3 x(3\cos^2 x - 5)$.

475. $y = \frac{(\operatorname{tg}^2 x - 1)(\operatorname{tg}^4 x + 10\operatorname{tg}^2 x + 1)}{3\operatorname{tg}^3 x}$.

476. $y = \operatorname{tg}^2 5x$.

477. $y = \frac{1}{2}\sin(x^2)$.

478. $y = \sin^2(t^3)$.

479. $y = 3\sin x\cos^2 x + \sin^3 x$.

480. $y = \frac{1}{3}\operatorname{tg}^3 x - \operatorname{tg} x + x$.

481. $y = -\frac{\cos x}{3\sin^3 x} + \frac{4}{3}\operatorname{ctg} x$.

482. $y = \sqrt{\alpha\sin^2 x + \beta\cos^2 x}$.

483. $y = \arcsin x^2 + \arccos x^2$.

484. $y = \frac{1}{2}(\arcsin x)^2 \arccos x$.

485. $y = \arcsin\frac{x^2-1}{x^2}$.

486. $y = \arcsin\frac{x}{\sqrt{1+x^2}}$.

487. $y = \frac{\arccos x}{\sqrt{1-x^2}}$.

488. $y = \frac{1}{\sqrt{b}}\arcsin\left(x\sqrt{\frac{b}{a}}\right)$.

489. $y = \sqrt{a^2-x^2} + a\arcsin\frac{x}{a}$.

490. $y = x\sqrt{a^2-x^2} + a^2\arcsin\frac{x}{a}$.

491. $y = \arcsin(1-x) + \sqrt{2x-x^2}$.

492. $y = \left(x-\frac{1}{2}\right)\arcsin\sqrt{x} + \frac{1}{2}\sqrt{x-x^2}$.

493. $y = \ln(\arcsin 5x)$.

494. $y = \arcsin(\ln x)$.

495. $y = \operatorname{arctg}\frac{x\sin\alpha}{1-x\cos\alpha}$.

496. $y = \frac{2}{3}\operatorname{arctg}\frac{5\operatorname{tg}\frac{x}{2}+4}{3}$.

497. $y = 3b^2\operatorname{arctg}\sqrt{\frac{x}{b-x}} - (3b+2x)\sqrt{bx-x^2}$.

498. $y = -\sqrt{2}\operatorname{arcctg}\frac{\operatorname{tg} x}{\sqrt{2}} - x$.

499. $y = \sqrt{e^{ax}}$.

500. $y = e^{\sin^2 x}$.

501. $F(x) = (2m\,a^{mx} + b)^p$.

502. $F(t) = e^{\alpha t}\cos\beta t$.

503. $y = \frac{(\alpha\sin\beta x - \beta\cos\beta x)e^{\alpha x}}{\alpha^2+\beta^2}$.

504. $y = \frac{1}{10}e^{-x}(3\sin 3x - \cos 3x)$.

505. $y = x^n a^{-x^2}$.

506. $y = \sqrt{\cos x}\,a^{\sqrt{\cos x}}$.

507. $y = 3^{\operatorname{ctg}\frac{1}{x}}$.

508. $y = \ln(ax^2+bx+c)$.

509. $y = \ln(x+\sqrt{a^2+x^2})$.

510. $y = x - 2\sqrt{x} + 2\ln(1+\sqrt{x})$.

511. $y = \ln(a+x+\sqrt{2ax+x^2})$.

512. $y = \dfrac{1}{\ln^2 x}$.

513. $y = \ln\cos\dfrac{x-1}{x}$.

514*. $y = \ln\dfrac{(x-2)^5}{(x+1)^3}$.

515. $y = \ln\dfrac{(x-1)^3(x-2)}{x-3}$.

516. $y = -\dfrac{1}{2\sin^2 x} + \ln \operatorname{tg} x$.

517. $y = \dfrac{x}{2}\sqrt{x^2-a^2} - \dfrac{a^2}{2}\ln(x+\sqrt{x^2-a^2})$.

518. $y = \ln\ln(3-2x^3)$.

519. $y = 5\ln^3(ax+b)$.

520. $y = \ln\dfrac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}-x}$.

521. $y = \dfrac{m}{2}\ln(x^2-a^2) + \dfrac{n}{2a}\ln\dfrac{x-a}{x+a}$.

522. $y = x\cdot\sin\left(\ln x - \dfrac{\pi}{4}\right)$.

523. $y = \dfrac{1}{2}\ln\operatorname{tg}\dfrac{x}{2} - \dfrac{1}{2}\dfrac{\cos x}{\sin^2 x}$.

524. $f(x) = \sqrt{x^2+1} - \ln\dfrac{1+\sqrt{x^2+1}}{x}$.

525. $y = \dfrac{1}{3}\ln\dfrac{x^2-2x+1}{x^2+x+1}$.

526. $y = 2^{\arcsin 3x} + (1-\arccos 3x)^2$.

527. $y = 3^{\frac{\sin ax}{\cos bx}} + \dfrac{1}{3}\dfrac{\sin^3 ax}{\cos^3 bx}$.

528. $y = \dfrac{1}{\sqrt{3}}\ln\dfrac{\operatorname{tg}\dfrac{x}{2}+2-\sqrt{3}}{\operatorname{tg}\dfrac{x}{2}+2+\sqrt{3}}$.

529. $y = \operatorname{arctg}\ln x$.

530. $y = \ln\arcsin x + \dfrac{1}{2}\ln^2 x + \arcsin\ln x$.

531. $y = \operatorname{arctg}\ln\dfrac{1}{x}$.

532. $y = \dfrac{\sqrt{2}}{3}\operatorname{arctg}\dfrac{x}{\sqrt{2}} + \dfrac{1}{6}\ln\dfrac{x-1}{x+1}$.

533. $y = \ln\dfrac{1+\sqrt{\sin x}}{1-\sqrt{\sin x}} + 2\operatorname{arctg}\sqrt{\sin x}$.

534. $y = \dfrac{3}{4}\ln\dfrac{x^2+1}{x^2-1} + \dfrac{1}{4}\ln\dfrac{x-1}{x+1} + \dfrac{1}{2}\operatorname{arctg} x$.

535. $f(x) = \dfrac{1}{3}\ln(1+x) - \dfrac{1}{6}\ln(x^2-x+1) + \dfrac{1}{\sqrt{3}}\operatorname{arctg}\dfrac{2x-1}{\sqrt{3}}$.

536. $f(x) = \dfrac{x\arcsin x}{\sqrt{1-x^2}} + \ln\sqrt{1-x^2}$.

537. $y = \operatorname{sh}^3 2x$.

538. $y = e^{\alpha x}\operatorname{ch}\beta x$.

539. $y = \operatorname{th}^3 2x$.

540. $y = \ln\operatorname{sh} 2x$.

541. $y = \operatorname{Arsh}\dfrac{x^2}{a^2}$.

542. $y = \operatorname{Arch}\ln x$.

543. $y = \operatorname{Arth}(\operatorname{tg} x)$.

544. $y = \operatorname{Arcth}(\sec x)$.

545. $y = \operatorname{Arth}\dfrac{2x}{1+x^2}$.

546. $y = \dfrac{1}{2}(x^2-1)\operatorname{Arth} x + \dfrac{1}{2}x$.

547. $y = \left(\dfrac{1}{2}x^2 + \dfrac{1}{4}\right)\operatorname{Arsh} x - \dfrac{1}{4}x\sqrt{1+x^2}$.

548. Izračunajte y', ako je

a) $y = |x|$; b) $y = x|x|$.

Konstruirajte grafove funkcija y i y'.

549. Izračunajte y', ako je

$$y = \ln|x| \quad (x \neq 0).$$

550. Izračunajte $f'(x)$, ako je

$$f(x) = \begin{cases} 1-x & \text{za } x \leqslant 0, \\ e^{-x} & \text{za } x > 0. \end{cases}$$

551. Izračunajte $f'(0)$, ako je

$$f(x) = e^{-x}\cos 3x.$$

Rješenje. $f'(x) = e^{-x}(-3\sin 3x) - e^{-x}\cos 3x$; $f'(0) = e^0(-3\sin 0) - e^0\cos 0 = -1$.

552. $f(x) = \ln(1+x) + \arcsin\dfrac{x}{2}$. Izračunajte $f'(1)$.

553. $y = \operatorname{tg}^3\dfrac{\pi x}{6}$. Izračunajte $\left(\dfrac{dy}{dx}\right)_{x=2}$.

554. Izračunajte $f'_+(0)$ i $f'_-(0)$ za funkcije:

a) $f(x) = \sqrt{\sin(x^2)}$;

b) $f(x) = \arcsin\dfrac{a^2-x^2}{a^2+x^2}$;

c) $f(x) = \dfrac{x}{1+e^{\frac{1}{x}}}$, $x \neq 0$; $f(0) = 0$;

d) $f(x) = x^2\sin\dfrac{1}{x}$, $x \neq 0$; $f(0) = 0$;

e) $f(x) = x\sin\dfrac{1}{x}$, $x \neq 0$; $f(0) = 0$.

361. $x_1=0$; $x_2=3$. *Uputa.* Jednadžba $f'(x)=f(x)$ za zadanu funkciju ima oblik $3x^2=x^3$.

362. 30 m/s. **363.** 1,2. **364.** -1. **365.** $f'(x_0)=\dfrac{-1}{x_0^2}$.

366. -1; 2; $\operatorname{tg}\varphi=3$. *Uputa.* Koristimo se rezultatima primjera 3 i zadatka 365.

367. *Rješenje.* a) $f'(0)=\lim\limits_{\Delta x\to 0}\dfrac{\sqrt[3]{(\Delta x)^2}}{\Delta x}=\lim\limits_{\Delta x\to 0}\dfrac{1}{\sqrt[3]{\Delta x}}=\infty$;

b) $f'(1)=\lim\limits_{\Delta x\to 0}\dfrac{\sqrt[5]{1+\Delta x}-1}{\Delta x}=\lim\limits_{\Delta x\to 0}\dfrac{1}{\sqrt[5]{(\Delta x)^4}}=+\infty$;

c) $f'-\left(\dfrac{2k+1}{2}\pi\right)=\lim\limits_{\Delta x\to -0}\dfrac{\left|\cos\left(\dfrac{2k+1}{2}\pi+\Delta x\right)\right|}{\Delta x}=\lim\limits_{\Delta x\to -0}\dfrac{|\sin\Delta x|}{\Delta x}=-1$;

$f'+\left(\dfrac{2k+1}{2}\right)=\lim\limits_{\Delta x\to +0}\dfrac{|\sin\Delta x|}{\Delta x}=1$.

368. $5x^4-12x^2+2$. **369.** $-\dfrac{1}{3}+2x-2x^3$.

370. $2ax+b$. **371.** $-\dfrac{15x^2}{a}$.

372. $mat^{m-1}+b(m+n)\,t^{m+n-1}$. **373.** $\dfrac{6ax^5}{\sqrt{a^2+b^2}}$.

374. $-\dfrac{\pi}{x^2}$. **375.** $2x^{-\frac{1}{3}}-5x^{\frac{3}{2}}-3x^{-4}$.

376. $\dfrac{8}{3}x^{\frac{5}{3}}$. *Uputa.* $y=x^2x^{\frac{2}{3}}=x^{\frac{8}{3}}$. **377.** $\dfrac{4b}{3x^2\sqrt[3]{x}}-\dfrac{2a}{3x\sqrt[3]{x^2}}$.

378. $\dfrac{bc-ad}{(c+dx)^2}$. **379.** $\dfrac{-2x^2-6x+25}{(x^2-5x+5)^2}$.

380. $\dfrac{1-4x}{x^2(2x-1)^2}$. **381.** $\dfrac{1}{\sqrt{z}\,(1-\sqrt{z})^2}$.

382. $5\cos x-3\sin x$. **383.** $\dfrac{4}{\sin^2 2x}$.

384. $\dfrac{-2}{(\sin x-\cos x)^2}$. **385.** $t^2\sin t$.

386. $y'=0$. **387.** $\operatorname{ctg} x-\dfrac{x}{\sin^2 x}$.

388. $\arcsin x+\dfrac{x}{\sqrt{1-x^2}}$. **389.** $x\operatorname{arctg} x$.

390. $x^6e^x(x+7)$. **391.** xe^x.

392. $e^x\dfrac{x-2}{x^3}$. **393.** $\dfrac{5x^4-x^5}{e^x}$.

394. $e^x(\cos x-\sin x)$. **395.** x^2e^x.

396. $e^x\left(\arcsin+\dfrac{1}{\sqrt{1-x^2}}\right)$. **397.** $\dfrac{x(2\ln x-1)}{\ln^2 x}$.

398. $3x^2\ln x$. **399.** $\dfrac{2}{x}+\dfrac{\ln x}{x^2}-\dfrac{2}{x^2}$.

400. $\dfrac{2\ln x}{x\ln 10}-\dfrac{1}{x}$. **401.** $\operatorname{sh} x+x\operatorname{ch} x$.

402. $\dfrac{2x\operatorname{ch} x-x^2\operatorname{sh} x}{\operatorname{ch}^2 x}$. **403.** $-\operatorname{th}^2 x$.

404. $\dfrac{-3(x\ln x+\operatorname{sh} x\operatorname{ch} x)}{x\ln^2 x\cdot\operatorname{sh}^2 x}$. **405.** $\dfrac{-2x^2}{1-x^4}$.

406. $\dfrac{1}{\sqrt{1-x^2}}\operatorname{Arsh} x+\dfrac{1}{\sqrt{1+x^2}}\arcsin x$. **407.** $\dfrac{x-\sqrt{x^2-1}\operatorname{Arch} x}{x^2\sqrt{x^2-1}}$.

408. $\dfrac{1+2x\operatorname{Arcth} x}{(1-x^2)^2}$. **410.** $\dfrac{3a}{c}\left(\dfrac{ax+b}{c}\right)^2$.

411. $12ab+18b^2y$. **412.** $16x(3+2x^2)^3$.

413. $\dfrac{x^2-1}{(2x-1)^8}$. **414.** $\dfrac{-x}{\sqrt{1-x^2}}$.

415. $\dfrac{bx^2}{\sqrt[3]{(a+bx^3)^2}}$. **416.** $-\sqrt{\sqrt[3]{\dfrac{a^2}{x^2}}-1}$.

418. $\dfrac{1-\operatorname{tg}^2 x+\operatorname{tg}^4 x}{\cos^2 x}$. **419.** $\dfrac{-1}{2\sin^2 x\sqrt{\operatorname{ctg} x}}$.

420. $2-15\cos^2 x\sin x$. **421.** $\dfrac{-16\cos 2t}{\sin^3 2t}$. *Uputa.* $x=\sin^{-2}t+\cos^{-2}t$.

422. $\dfrac{\sin x}{(1-3\cos x)^3}$. **423.** $\dfrac{\sin^3 x}{\cos^4 x}$.

424. $\dfrac{3\cos x+2\sin x}{2\sqrt{15\sin x-10\cos x}}$. **425.** $\dfrac{2\cos x}{3\sqrt[3]{\sin x}}+\dfrac{3\sin x}{\cos^4 x}$.

426. $\dfrac{1}{2\sqrt{1-x^2}\sqrt{1+\arcsin x}}$. **427.** $\dfrac{1}{2(1+x^2)\sqrt{\operatorname{arctg} x}}-\dfrac{3(\arcsin x)^2}{\sqrt{1-x^2}}$.

428. $\dfrac{-1}{(1+x^2)(\operatorname{arctg} x)^2}$. **429.** $\dfrac{e^x+xe^x+1}{2\sqrt{xe^x+x}}$.

430. $\dfrac{2e^x-2^x\ln 2}{3\sqrt[3]{(2e^x-2^x+1)^2}}+\dfrac{5\ln^4 x}{x}$. **432.** $(2x-5)\times\cos(x^5-5x+1)-\dfrac{a}{x^2\cos^2\frac{a}{x}}$.

433. $-\alpha\sin(\alpha x+\beta)$. **434.** $\sin(2t+\varphi)$.

435. $-2\dfrac{\cos x}{\sin^3 x}$. **436.** $\dfrac{-1}{\sin^2\frac{x}{a}}$.

437. $x\cos 2x^2\sin 3x^2$. **439.** $\dfrac{-2}{x\sqrt{x^4-1}}$.

440. $\dfrac{-1}{2\sqrt{x-x^2}}$. **441.** $\dfrac{-1}{1+x^2}$.

442. $\dfrac{-1}{1+x^2}$.

443. $-10xe^{-x^2}$.

444. $-2x\,5^{-x^2}\ln 5$.

445. $2x\,10^{2x}(1+x\ln 10)$.

446. $\sin 2^t+2^t\,t\cos 2^t\ln 2$.

447. $\dfrac{-e^x}{\sqrt{1-e^{2x}}}$.

448. $\dfrac{2}{2x+7}$.

449. $\operatorname{ctg} x\lg e$.

450. $\dfrac{-2x}{1-x^2}$

451. $\dfrac{2\ln x}{x}-\dfrac{1}{x\ln x}$.

452. $\dfrac{(e^x+5\cos x)\sqrt{1-x^2}-4}{(e^x+5\sin x-4\arcsin x)\sqrt{1-x^2}}$.

453. $\dfrac{1}{(1+\ln^2 x)x}+\dfrac{1}{(1+x^2)\operatorname{arctg} x}$.

454. $\dfrac{1}{2x\sqrt{\ln x+1}}+\dfrac{1}{2(\sqrt{x}+x)}$.

455. *Rješenje.* $y'=(\sin^3 5x)'\cos^2\dfrac{x}{3}+\sin^3 5x\left(\cos^2\dfrac{x}{3}\right)'=3\sin^2 5x\cos 5x\cdot 5\cos^2\dfrac{x}{3}+$

$+\sin^3 5x\cdot 2\cos\dfrac{x}{3}\left(-\sin\dfrac{x}{3}\right)\dfrac{1}{3}=15\sin^2 5x\cos 5x\cos^2\dfrac{x}{3}-\dfrac{2}{3}\sin^3 5x\cos\dfrac{x}{3}\sin\dfrac{x}{3}$.

456. $\dfrac{4x+3}{(x-2)^3}$.

457. $\dfrac{x^2+4x-6}{(x-3)^5}$.

458. $\dfrac{x^7}{(1-x^2)^5}$.

459. $\dfrac{x-1}{x^2\sqrt{2x^2-2x+1}}$.

460. $\dfrac{1}{\sqrt{(a^2+x^2)^3}}$.

461. $\dfrac{x^2}{\sqrt{1+x^2)^5}}$.

462. $\dfrac{(1+\sqrt{x})^3}{\sqrt[3]{x}}$.

463. $x^6\sqrt[3]{(1+x^3)^2}$.

464. $\dfrac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}$.

465. $4x^3(a-2x^3)(a-5x^3)$.

466. $\dfrac{2abmnx^{n-1}(a+bx^n)^{m-1}}{(a-bx^n)^{m+1}}$.

467. $\dfrac{x^3-1}{(x+2)^6}$.

468. $\dfrac{a-3x}{2\sqrt{a-x}}$.

469. $\dfrac{3x^2+2(a+b+c)x+ab+bc+ac}{2\sqrt{(x+a)(x+b)(x+c)}}$.

470. $\dfrac{1+2\sqrt{y}}{6\sqrt{y}\sqrt[3]{(y+\sqrt{y})^2}}$.

471. $2(7t+4)\sqrt[3]{3t+2}$.

472. $\dfrac{y-a}{\sqrt{(2ay-y^2)^5}}$.

473. $\dfrac{1}{\sqrt{e^x+1}}$.

474. $\sin^3 x\cos^2 x$.

475. $\dfrac{1}{\sin^4 x\cos^4 x}$.

476. $10\operatorname{tg} 5x\sec^2 5x$.

477. $x\cos x^2$.

478. $3t^2\sin 2t^3$.

479. $3\cos x\cos 2x$.

480. $\operatorname{tg}^4 x$.

481. $\dfrac{\cos 2x}{\sin^4 x}$.

482. $\dfrac{(\alpha-\beta)\sin 2x}{2\sqrt{\alpha\sin^2 x+\beta\cos^2 x}}$.

483. 0.

484. $\dfrac{1}{2}\,\dfrac{\arcsin x(2\arccos x-\arcsin x)}{\sqrt{1-x^2}}$

485. $\dfrac{2}{x\sqrt{2x^2-1}}$.

486. $\dfrac{1}{1+x^2}$.

487. $\dfrac{x\arccos x-\sqrt{1-x^2}}{(1-x^2)^{3/2}}$.

488. $\dfrac{1}{\sqrt{a-bx^2}}$.

489. $\sqrt{\dfrac{a-x}{a+x}}$ $(a>0)$.

490. $2\sqrt{a^2-x^2}$ $(a>0)$.

491. $\dfrac{-x}{\sqrt{2x-x^2}}$.

492. $\arcsin\sqrt{x}$.

493. $\dfrac{5}{\sqrt{1-25x^2}\arcsin 5x}$.

494. $\dfrac{1}{x\sqrt{1-\ln^2 x}}$.

495. $\dfrac{\sin\alpha}{1-2x\cos\alpha+x^2}$.

496. $\dfrac{1}{5+4\sin x}$.

497. $4x\sqrt{\dfrac{x}{b-x}}$.

498. $\dfrac{\sin^2 x}{1+\cos^2 x}$.

499. $\dfrac{a}{2}\sqrt{e^{ax}}$.

500. $\sin 2xe^{\sin^2 x}$.

501. $2m^2p(2ma^{mx}+b)^{p-1}a^{mx}\ln a$.

502. $e^{\alpha t}(\alpha\cos\beta t-\beta\sin\beta t)$.

503. $e^{\alpha x}\sin\beta x$.

504. $e^{-x}\cos 3x$.

505. $x^{n-1}a^{-x^2}(n-2x^2\ln a)$.

506. $-\dfrac{1}{2}y\operatorname{tg} x(1+\sqrt{\cos x}\ln a)$.

507. $\dfrac{3^{\operatorname{ctg}\frac{1}{x}}\ln 3}{\left(x\sin\dfrac{1}{x}\right)^2}$.

508. $\dfrac{2ax+b}{ax^2+bx+c}$.

509. $\dfrac{1}{\sqrt{a^2+x^2}}$.

510. $\dfrac{\sqrt{x}}{1+\sqrt{x}}$.

511. $\dfrac{1}{\sqrt{2ax+x^2}}$.

512. $\dfrac{-2}{x\ln^3 x}$.

513. $-\dfrac{1}{x^2}\operatorname{tg}\dfrac{x-1}{x}$.

514. $\dfrac{2x+11}{x^2-x-2}$. *Uputa.* $y=5\ln(x-2)-3\ln(x+1)$.

515. $\dfrac{3x^2-16x+19}{(x-1)(x-2)(x-3)}$.

516. $\dfrac{1}{\sin^3 x\cos x}$.

517. $\sqrt{x^2-a^2}$.

518. $\dfrac{-6x^2}{(3-2x^3)\ln(3-2x^3)}$.

519. $\dfrac{15a\ln^2(ax+b)}{ax+b}$.

520. $\dfrac{2}{\sqrt{x^2+a^2}}$.

521. $\dfrac{mx+n}{x^2-a^2}$.

522. $\sqrt{2}\sin\ln x$.

523. $\dfrac{1}{\sin^3 x}$.

524. $\dfrac{\sqrt{1+x^2}}{x}$.

525. $\dfrac{x+1}{x^3-1}$.

526. $\dfrac{3}{\sqrt{1-9x^2}}[2^{\arcsin 3x}\ln 2+2(1-\arccos 3x)]$.

527. $\left(3^{\frac{\sin ax}{\cos bx}}\ln 3+\dfrac{\sin^2 ax}{\cos^2 bx}\right)\cdot\dfrac{a\cos ax\cos bx+b\sin ax\sin bx}{\cos^2 bx}$.

528. $\dfrac{1}{1+2\sin x}$.

529. $\dfrac{1}{x(1+\ln^2 x)}$.

530. $\dfrac{1}{\sqrt{1-x^2}\arcsin x}+\dfrac{\ln x}{x}+\dfrac{1}{x\sqrt{1-\ln^2 x}}$.

531. $-\dfrac{1}{x(1+\ln^2 x)}$.

532. $\dfrac{x^2}{x^4+x^2-2}$.

533. $\dfrac{1}{\cos x\sqrt{\sin x}}$.

534. $\dfrac{x^2-3x}{x^4-1}$.

535. $\dfrac{1}{1+x^3}$.

536. $\dfrac{\arcsin x}{(1-x^2)^{3/2}}$.

537. $6\operatorname{sh}^2 2x\cdot\operatorname{ch} 2x$.

538. $e^{\alpha x}(\alpha\operatorname{ch}\beta x+\beta\operatorname{sh}\beta x)$.

539. $6\operatorname{th}^2 2x(1-\operatorname{th}^2 2x)$.